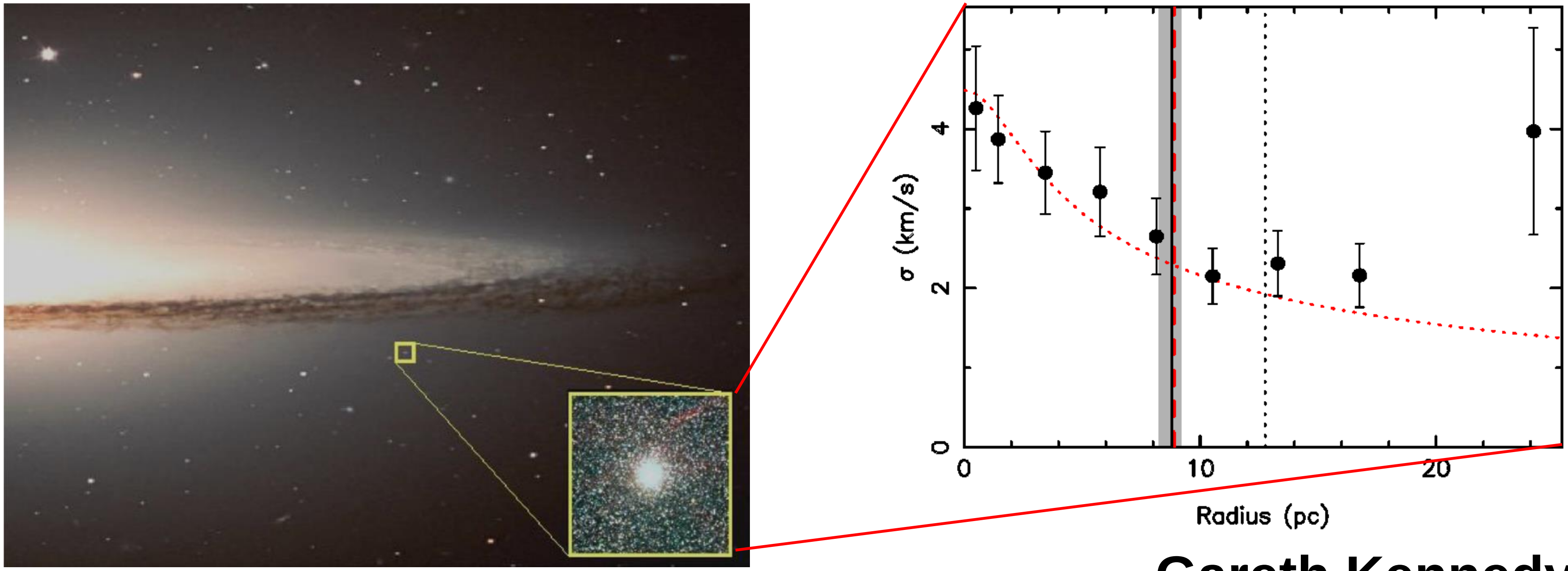
The role of three-body stability in tidally interacting globular clusters



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Overview

- Galactic globular cluster system
- Stability boundary inside a cluster beyond which stars are unstable to escape from the cluster
- Application to velocity dispersion observations for the Milky Way GC system
- Comparison between the stability boundary method (based on Newtonian dynamics) and MOND in the context of flattening of velocity dispersions

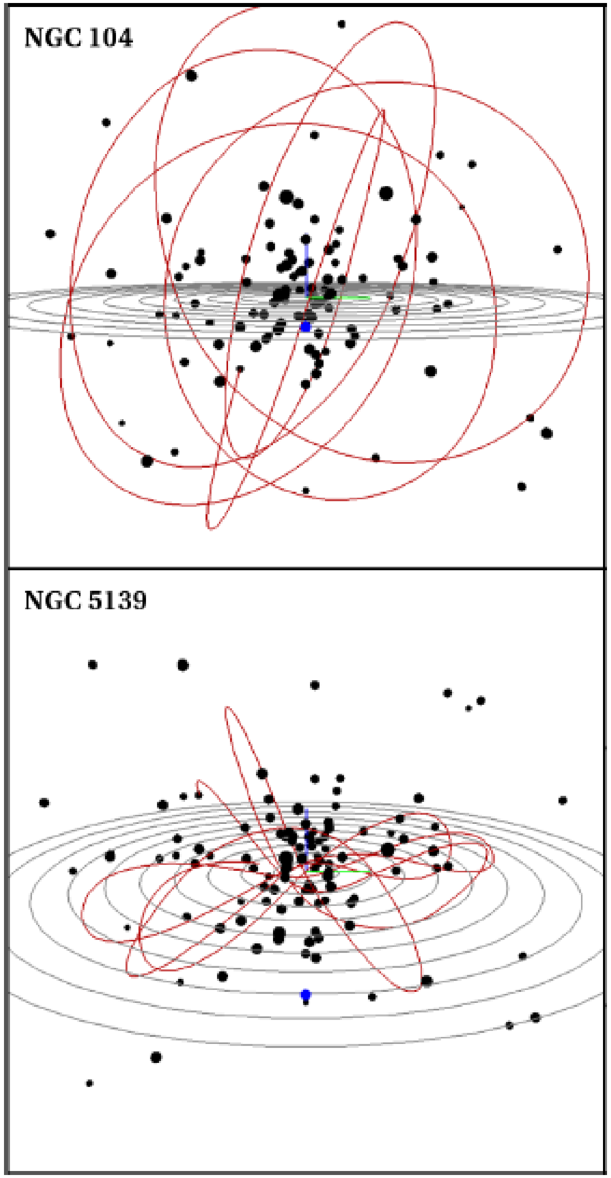
- **Based on astroph: 1108.5241 and 1108.5242 (resubmitted after review 6/12/2013)**

Globular cluster orbits

- Globular clusters are not isolated systems
- The Galaxy has an effect, even if they are not being actively tidally disrupted
- To investigate effect of tides I looked at the orbits of 15 GCs with observed velocity components (and published velocity dispersions vs. radius)
- GC-galaxy orbits were determined from these and approximated by Keplerian orbital elements so that a 3-body stability analysis could be applied

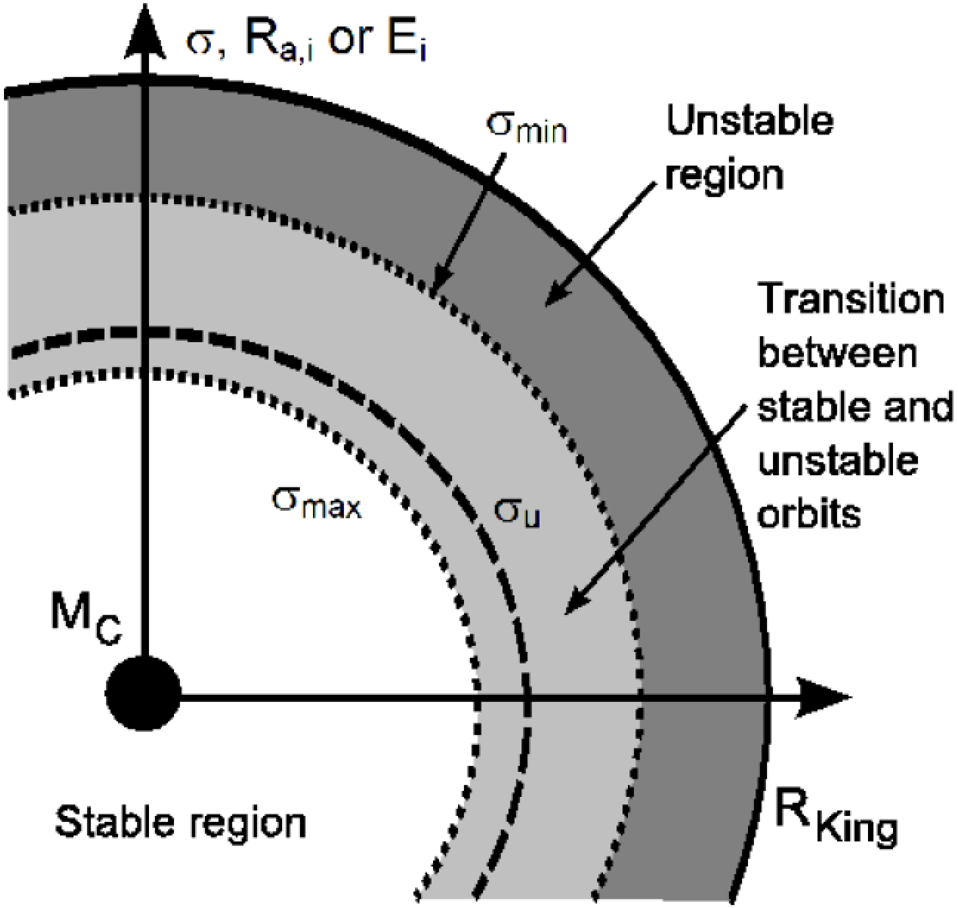
Galactic Globular Cluster System

- Galactic potential of Fellhauer et al. 2007 is used and the cluster orbits are integrated back in time
- Physical positions and velocities from observations, but subject to large uncertainties in tangential velocity and distances
- Minimum/maximum distances from calculated orbit are used to determine peri/apogalacticon
- Use observational errors to generate multiple (10^3) positions and velocities for each GC



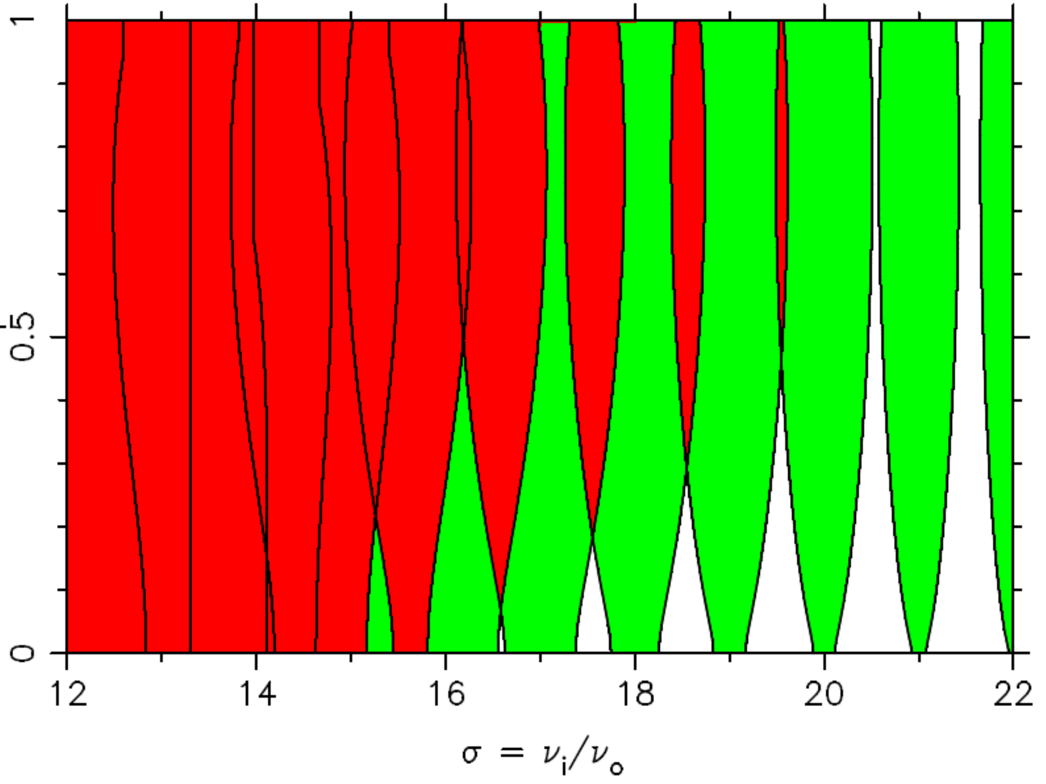
Stability boundary

- Wish to find a radius such that all stars on exterior orbits will be unstable to escape from the cluster
- Will use the stability of the general three-body problem to determine this by treating the star, cluster and galaxy as point-mass particles
- Stability boundary given by averaging over star-cluster orbits



Calculating the stability boundary

- Use Rosemary Mardling's stability criterion with additional terms for inclined orbits
- System is predicted to be unstable if two adjacent resonances (green) with a period ratio of n:1 overlap (red)



- In the context of a star-cluster centre-galaxy system then unstable means that the star will ***eventually*** escape the system
- Timescale of approx. 10 GC-galaxy orbits

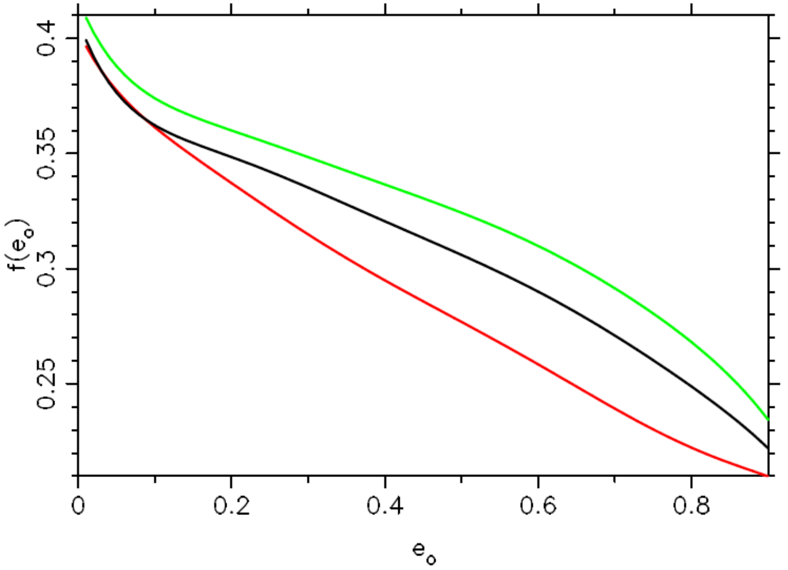
Dependence on eccentricity

- Write the stability boundary in the form

$$r_t = R_p \left(\frac{M_C}{M_G} \right)^{1/3} f(e)$$

where R_p is the perigalacticon, M_C is the cluster mass and M_G is the mass of the galaxy particle.

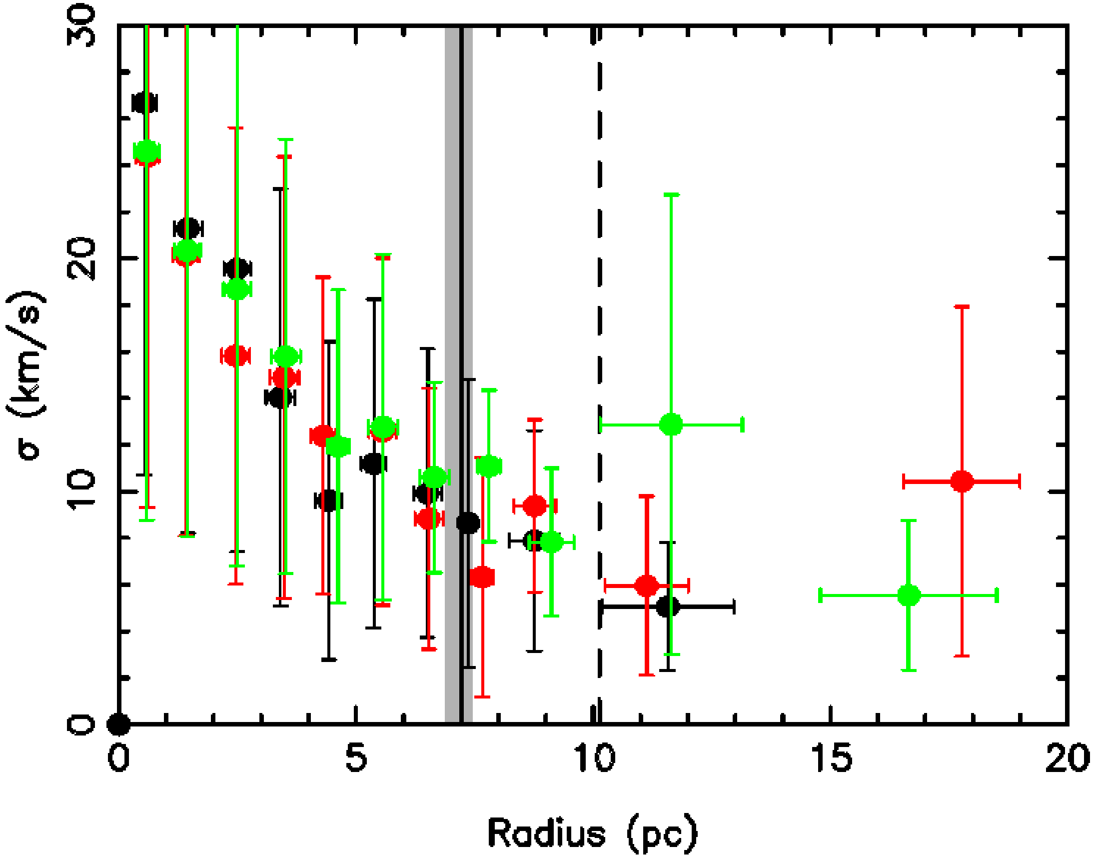
- The dependence of f(e) on the cluster-galaxy orbital eccentricity is shown to the right. The curves show the min/max and r_{chaos} values



- Tidal radius (King 1962) is: $f(e) = 0.7\ (3 + e)^{-1/3}$ which varies from 0.48 to 0.44 at e = 0.9

Effect of unstable orbits

Results from a simple cluster model used to investigate the effect of particles on unstable orbits on the velocity dispersion



Velocity dispersion profile after 10 (black), 20 (red) and 30 (green) cluster-galaxy orbits is shown. The transition from stable inner to unstable outer orbits is shaded and the indicative radius (r_{chaos}) is shown as a vertical line. The dashed line shows the King radius.

Velocity dispersions

- Equilibrium model based on Newtonian dynamics

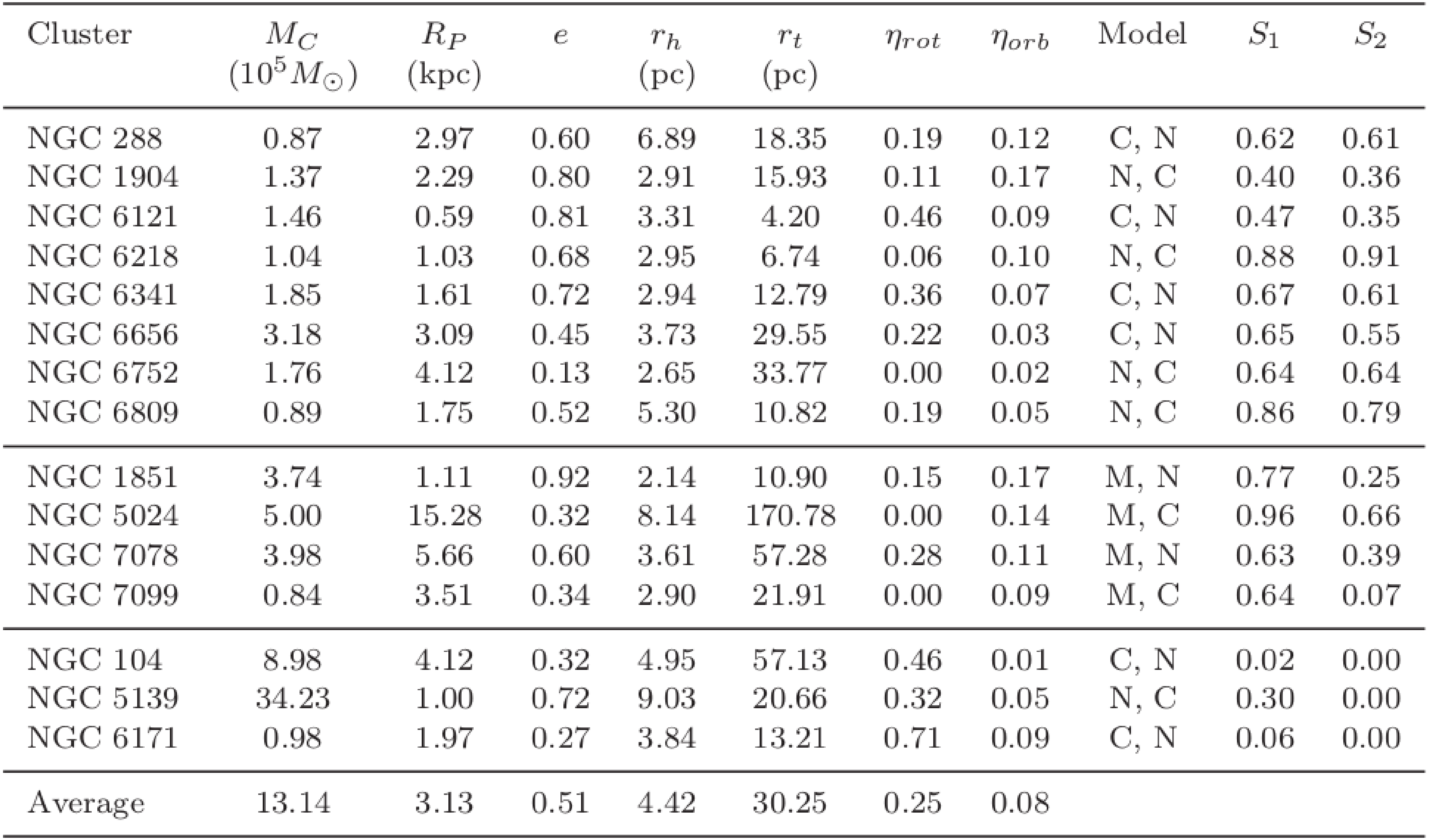
$$\sigma^2(R) = \frac{\sigma_0^2}{\sqrt{1+\frac{r^2}{r_{1/2}^2}}} \quad \text{where} \quad M_C = \frac{64\sigma_0^2 r_{1/2}}{3\pi G}$$

  links the cluster mass with the central σ
- Observations of flattening velocity dispersion at large distances from the cluster centre
- Possible explanations:
  - Tidal interactions with the galaxy
  - Breakdown of Newtonian dynamics
  - **Chaotic orbits in outer regions**

Velocity dispersions

- 15 clusters have been chosen with good radial coverage of the velocity dispersion and with all of the cluster velocity components published
- A fit to the central velocity dispersion (where it is Newtonian) is used to determine the cluster mass
- Used Bayesian analysis to compare models
- Clusters divided by preferred model:
  - Newtonian (**C** = chaos or **N** = no flattening): NGC 6341
  - MOND (**M**) candidates: NGC 1851
  - No preferred model: NGC 6171

Comparison summary



Cluster	M_C ($10^5 M_\odot$)	R_P (kpc)	e	r_h (pc)	r_t (pc)	η_{rot}	η_{orb}	Model	S_1	S_2
NGC 288	0.87	2.97	0.60	6.89	18.35	0.19	0.12	C, N	0.62	0.61
NGC 1904	1.37	2.29	0.80	2.91	15.93	0.11	0.17	N, C	0.40	0.36
NGC 6121	1.46	0.59	0.81	3.31	4.20	0.46	0.09	C, N	0.47	0.35
NGC 6218	1.04	1.03	0.68	2.95	6.74	0.06	0.10	N, C	0.88	0.91
NGC 6341	1.85	1.61	0.72	2.94	12.79	0.36	0.07	C, N	0.67	0.61
NGC 6656	3.18	3.09	0.45	3.73	29.55	0.22	0.03	C, N	0.65	0.55
NGC 6752	1.76	4.12	0.13	2.65	33.77	0.00	0.02	N, C	0.64	0.64
NGC 6809	0.89	1.75	0.52	5.30	10.82	0.19	0.05	N, C	0.86	0.79
NGC 1851	3.74	1.11	0.92	2.14	10.90	0.15	0.17	M, N	0.77	0.25
NGC 5024	5.00	15.28	0.32	8.14	170.78	0.00	0.14	M, C	0.96	0.66
NGC 7078	3.98	5.66	0.60	3.61	57.28	0.28	0.11	M, N	0.63	0.39
NGC 7099	0.84	3.51	0.34	2.90	21.91	0.00	0.09	M, C	0.64	0.07
NGC 104	8.98	4.12	0.32	4.95	57.13	0.46	0.01	C, N	0.02	0.00
NGC 5139	34.23	1.00	0.72	9.03	20.66	0.32	0.05	N, C	0.30	0.00
NGC 6171	0.98	1.97	0.27	3.84	13.21	0.71	0.09	C, N	0.06	0.00
Average	13.14	3.13	0.51	4.42	30.25	0.25	0.08			

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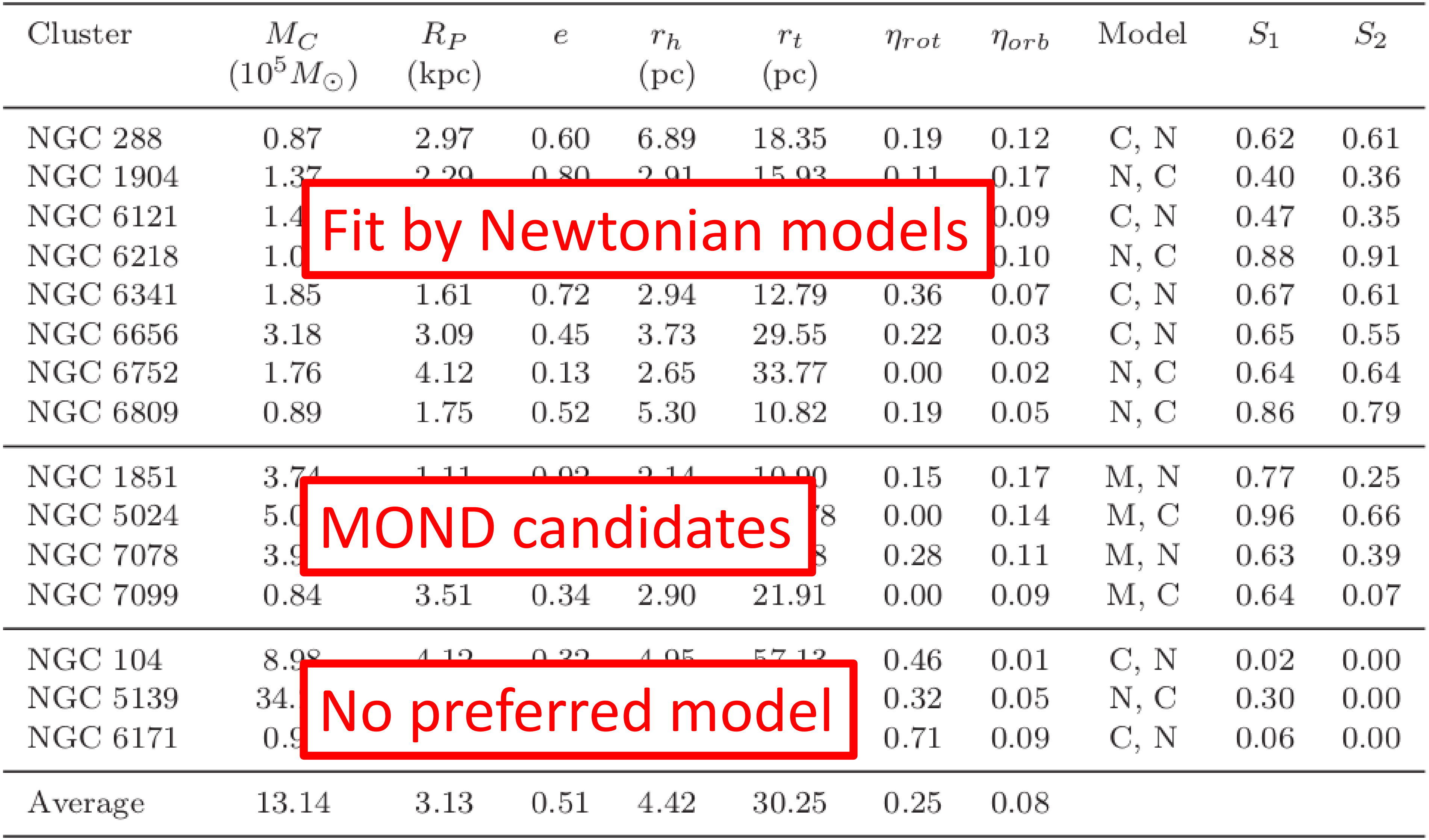

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NGC 5024	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.14	M, C	0.96	0.66
NGC 7078	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.28	0.11	M, N	0.63	0.39
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NGC 6171	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.71	0.09	C, N	0.06	0.00
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Fit by Newtonian models

MOND candidates

No preferred model

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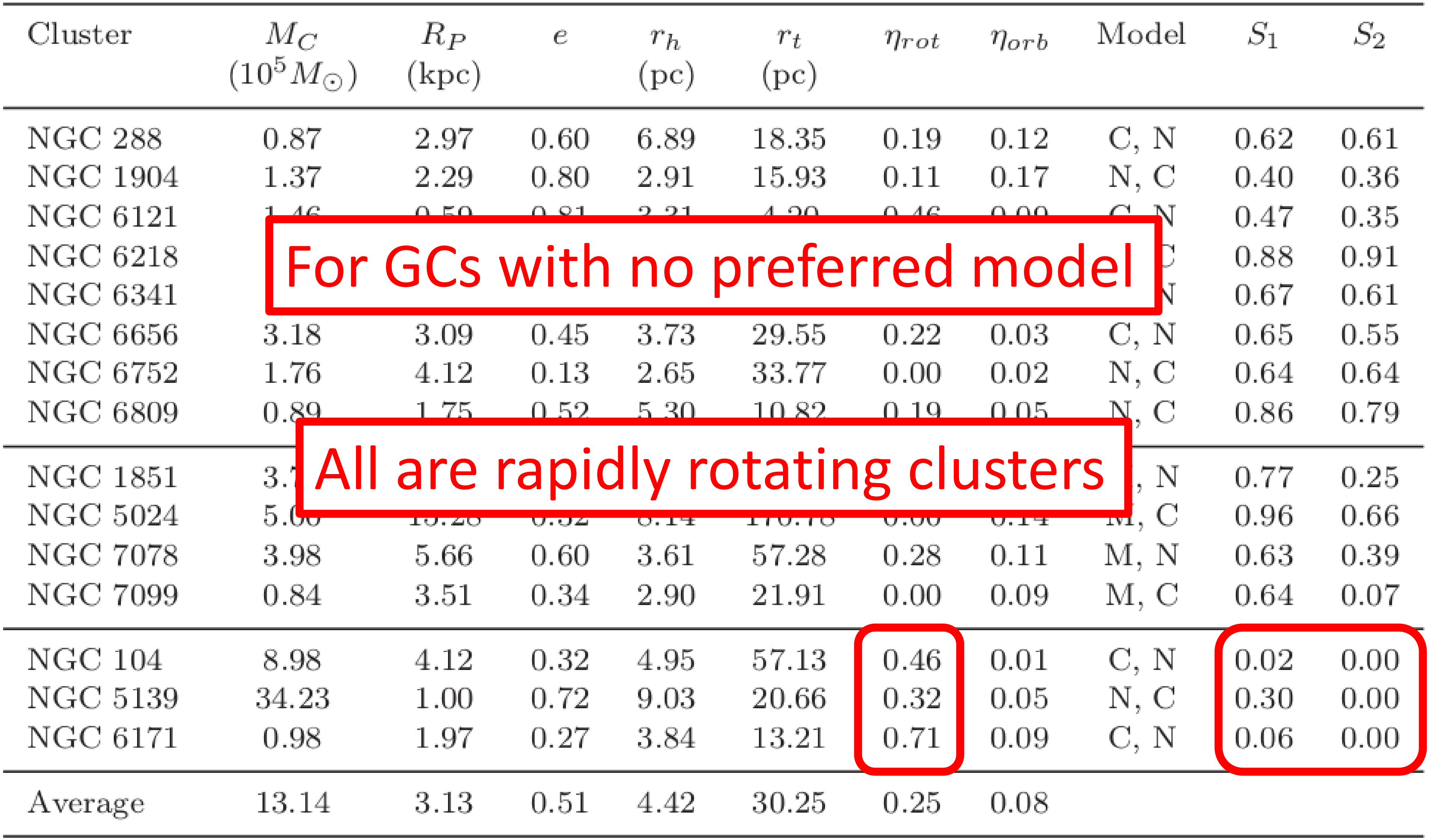

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For GCs with no preferred model

All are rapidly rotating clusters

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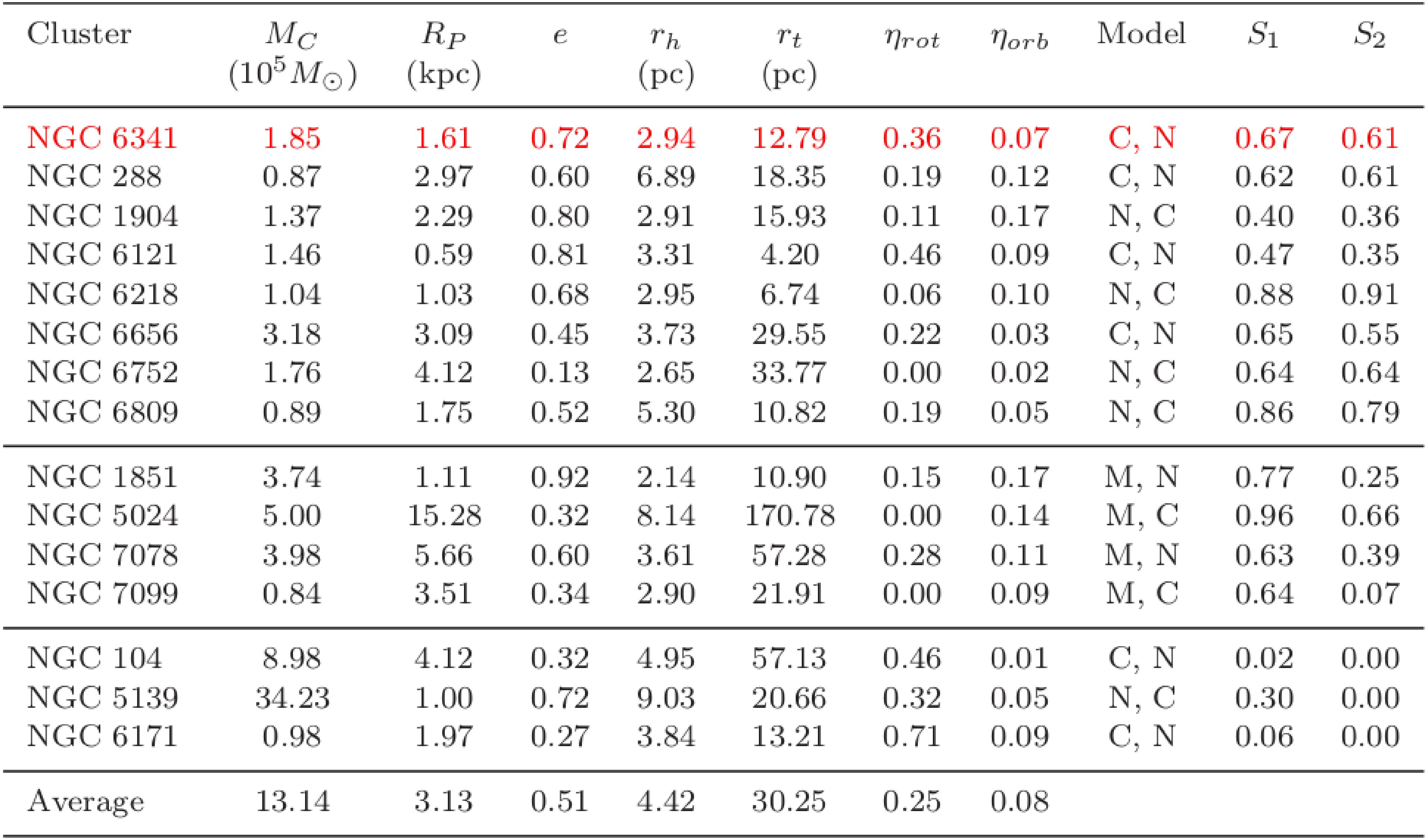

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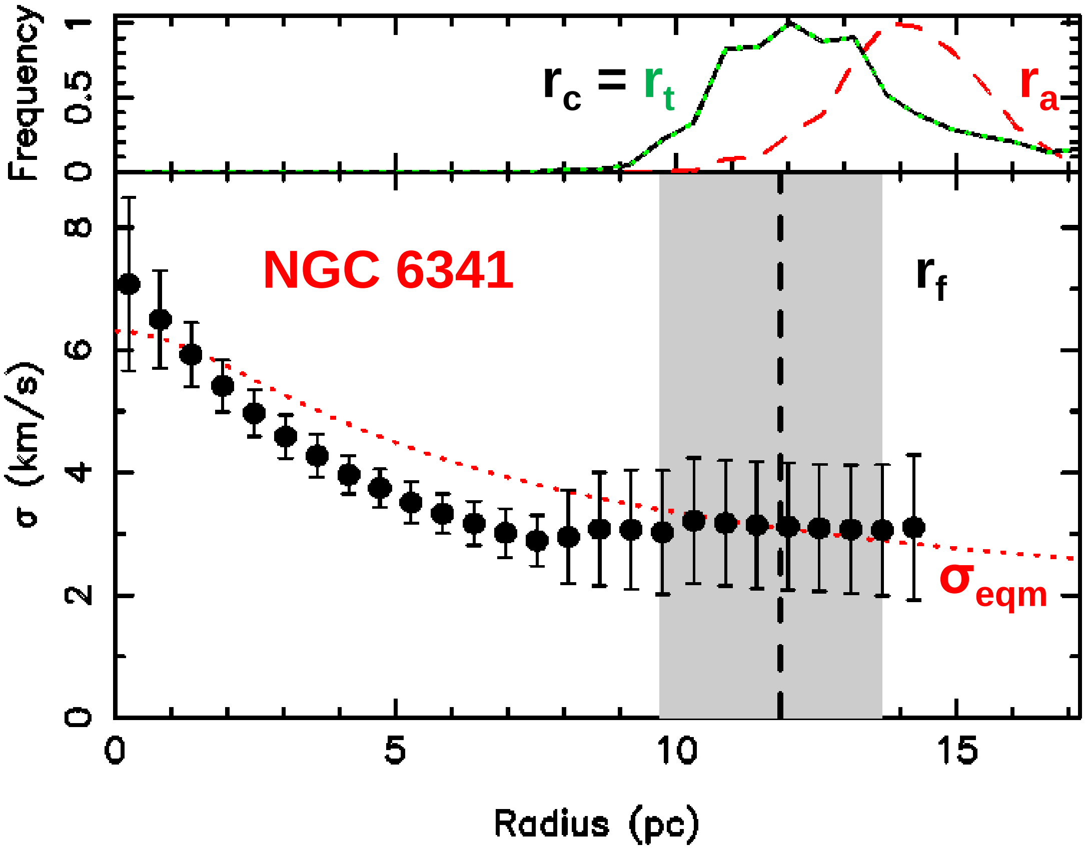
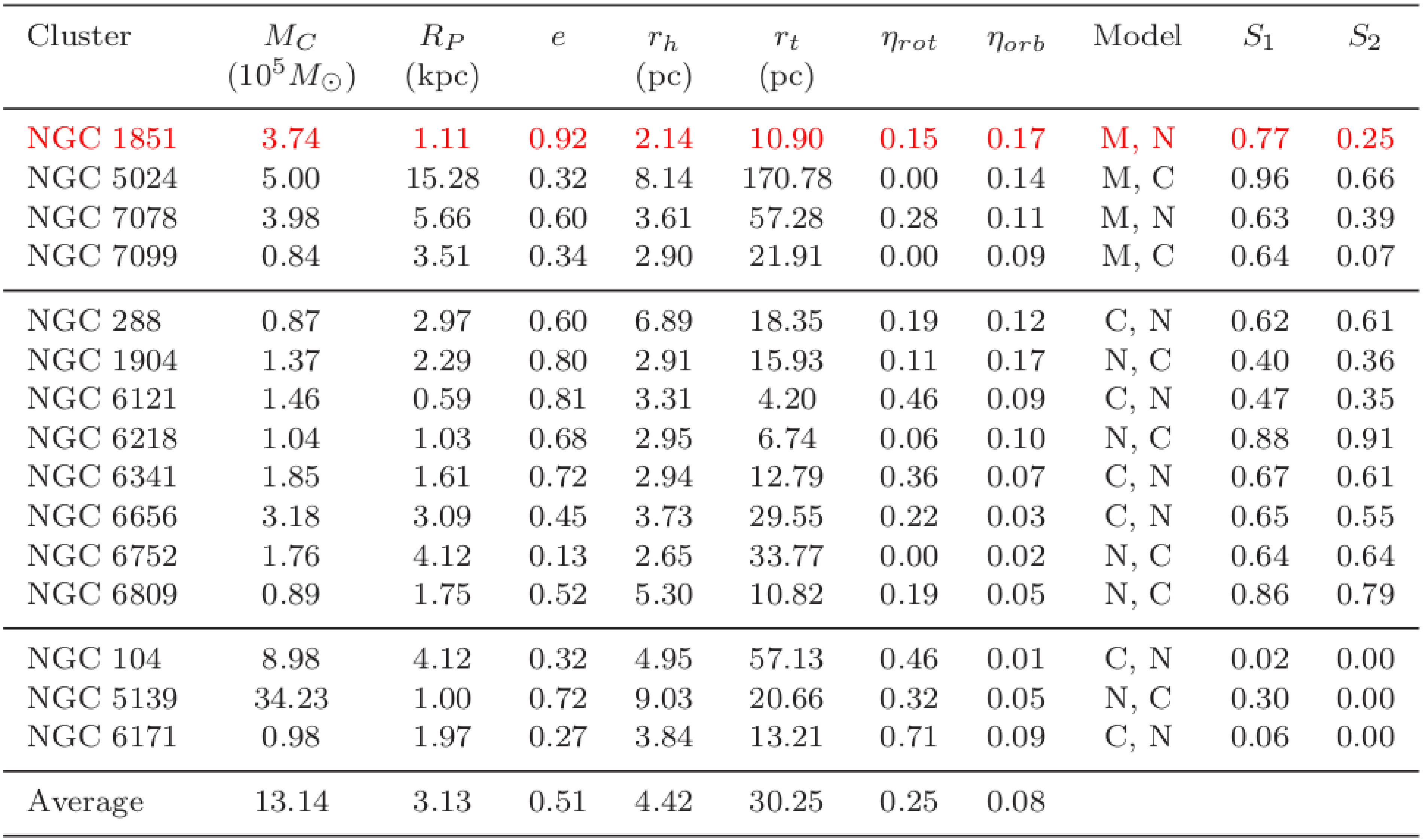


MOND candidates



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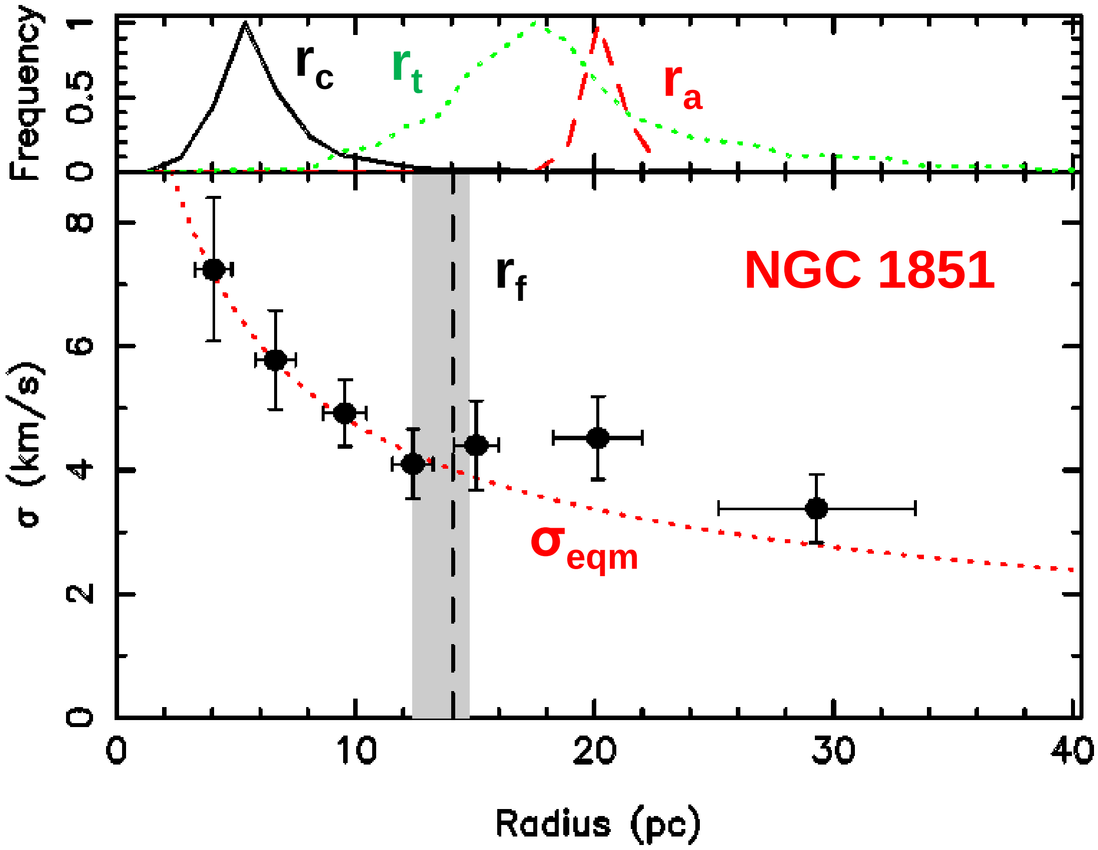
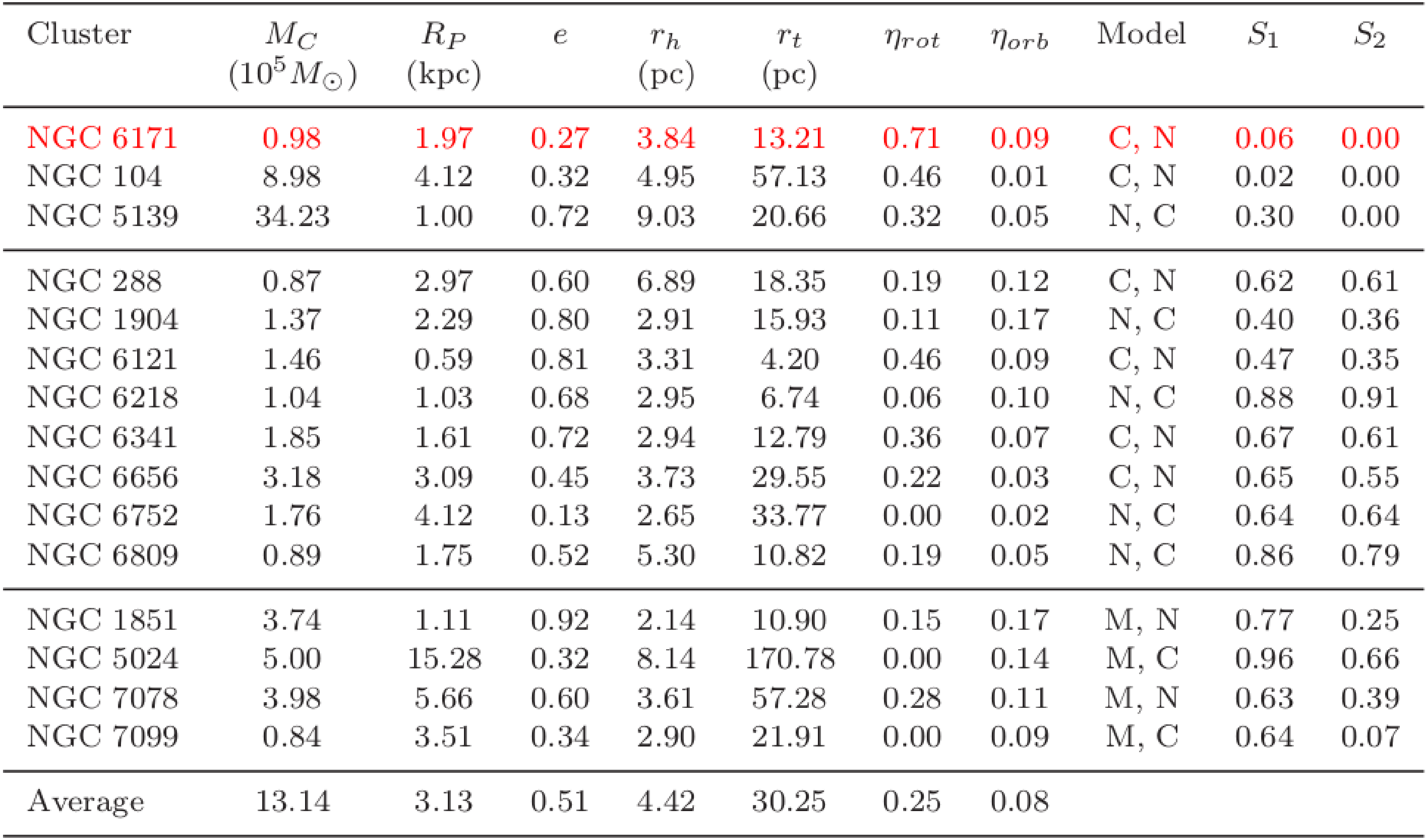


No preferred model



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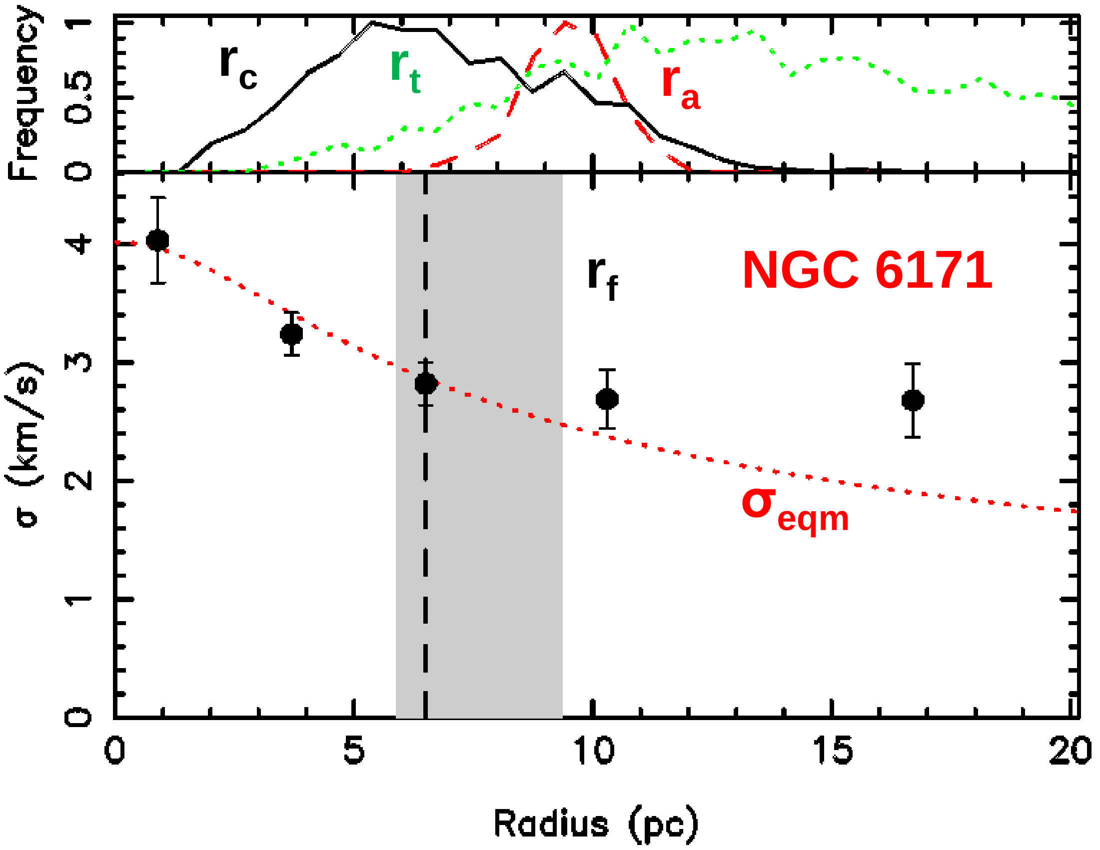
No preferred model

Cluster	M_C ($10^5 M_\odot$)	R_P (kpc)	e	r_h (pc)	r_t (pc)	η_{rot}	η_{orb}	Model	S_1	S_2
NGC 6171	0.98	1.97	0.27	3.84	13.21	0.71	0.09	C, N	0.06	0.00
NGC 104	8.98	4.12	0.							
NGC 5139	34.23	1.00	0.							
NGC 288	0.87	2.97	0.							
NGC 1904	1.37	2.29	0.							
NGC 6121	1.46	0.59	0.							
NGC 6218	1.04	1.03	0.							
NGC 6341	1.85	1.61	0.							
NGC 6656	3.18	3.09	0.							
NGC 6752	1.76	4.12	0.							
NGC 6809	0.89	1.75	0.							
NGC 1851	3.74	1.11	0.							
NGC 5024	5.00	15.28	0.							
NGC 7078	3.98	5.66	0.							
NGC 7099	0.84	3.51	0.							
Average	13.14	3.13	0.							



Conclusions

- **Flattening of the velocity dispersion of globular clusters is predicted to occur beyond a certain radius by consideration of three-body stability in Newtonian dynamics**
- This occurs in the outer regions of a cluster where two-body relaxation is (generally) negligible and in clusters which are not being strongly tidally disrupted
- **Predicted radius depends on the GC-galaxy orbit and not just on the cluster mass, which provides a way of distinguishing these predictions from MOND models**
- Additional observations of GC proper motions will provide a strong test for both of these models
- **Currently running n-body simulations to closer examine the effect in realistic galactic potentials**