Expansion methods in stellar dynamics

Yohai Meiron

Kavli Institute for Astronomy and Astrophysics
Peking University

5^{th} China-Korea workshop on stellar dynamics and gravitational waves

Beijing

December 12, 2013

Expansion methods in stellar dynamics

Yohai Meiron

Kavli Institute for Astronomy and Astrophysics
Peking University

PROGRESS REPORT

5th China-Korea workshop on stellar dynamics and gravitational waves

Beijing

December 12, 2013

Outline

- Motivation
- Theory
- Implementation & Examples
- Summary

Motivation

Two fast & approximate approaches

- The **Monte Carlo method** concerns itself with old, spherically symmetric clusters in dynamical equilibrium where evolution is dominated by relaxation. *Very good for galactic star clusters*
- **Expansion methods** can deal with non-equilibrium systems of any geometry, but assume a very long relaxation time. *Very good for galactic centers*

Could the two interface (possibly via AMUSE)?

Motivation

Two fast & approximate approaches

- The **Monte Carlo method** concerns itself with old, spherically symmetric clusters in dynamical equilibrium where evolution is dominated by relaxation. *Very good for galactic star clusters*
- **Expansion methods** can deal with non-equilibrium systems of any geometry, but assume a very long relaxation time. *Very good for galactic centers*

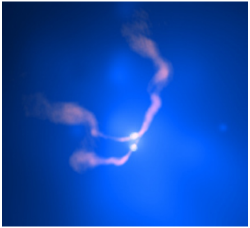
Maxwell Tsai's talk

Could the two interface (possibly via AMUSE)?

Motivation

More on galaxy centers

Binary supermassive black holes (BBHs)
in a galactic center environment. . .



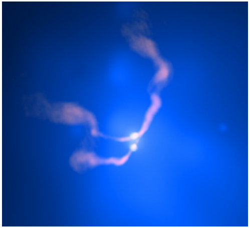
NGC 1128

Motivation

More on galaxy centers

Binary supermassive black holes (BBHs) in a galactic center environment...

- A tiny fraction of stars affects the BBH significantly via secular processes.
  very large N is needed to resolve this!



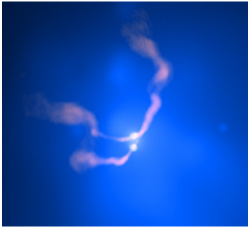
NGC 1128

Motivation

More on galaxy centers

Binary supermassive black holes (BBHs) in a galactic center environment. . .

- A tiny fraction of stars affects the BBH significantly via secular processes.
  very large N is needed to resolve this!
- The interesting physics is in the BBH–star interaction.



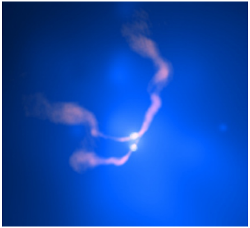
NGC 1128

Motivation

More on galaxy centers

Binary supermassive black holes (BBHs) in a galactic center environment. . .

- A tiny fraction of stars affects the BBH significantly via secular processes.
  very large N is needed to resolve this!
- The interesting physics is in the BBH–star interaction.
- The force due to other stars is *mostly* radial.*



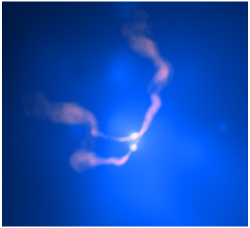
NGC 1128

Motivation

More on galaxy centers

Binary supermassive black holes (BBHs) in a galactic center environment. . .

- A tiny fraction of stars affects the BBH significantly via secular processes.
  very large N is needed to resolve this!
- The interesting physics is in the BBH–star interaction.
- The force due to other stars is *mostly* radial.*
- Relaxation time is very long.



NGC 1128

Why is the N-body problem a problem?

Because those are $3N$ coupled non-linear second order differential equations, where $N \approx 10^{11}$ in a galaxy.

$$\ddot{\mathbf{r}}_i = -G \sum_{i \neq j} \frac{m_j(\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

— — — — — — — — — — — — — — — — — —

Why is the N-body problem a problem?

Because those are $3N$ coupled non-linear second order differential equations, where $N \approx 10^{11}$ in a galaxy.

$$\ddot{\mathbf{r}}_i = -G \sum_{i \neq j} \frac{m_j (\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

— — — — — — — — — — — — — — — — — —

- N is smaller by orders of magnitude.
  - 2-body relaxation time $\propto N / \ln N$
  - DF not properly populated

Rough analogy: digital images

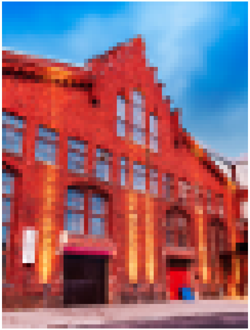
600×800.tiff file
1400 KB

"full" N-body

Rough analogy: digital images

600×800.tiff file
1400 KB

"full" N-body



60×80.tiff file
14 KB

smaller N

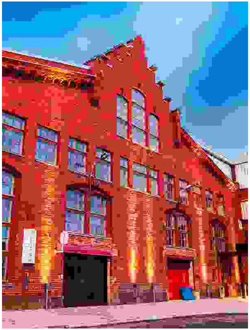
$\Delta \approx 36\%$

$\Delta = 100\%$ means white noise difference image; zero means identical.

Rough analogy: digital images

600×800.tiff file
1400 KB

"full" N-body



.jpg file
7 KB

low order

$\Delta \approx 33\%$

$\Delta = 100\%$ means white noise difference image; zero means identical.

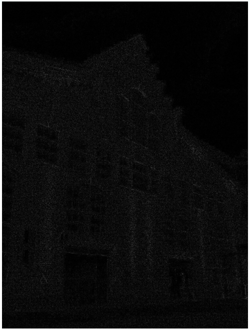
Rough analogy: digital images

600×800.tiff file
1400 KB

"full" N-body

.jpg file
223 KB

high order



$\Delta \approx 5\%$

$\Delta = 100\%$ means white noise difference image; zero means identical.

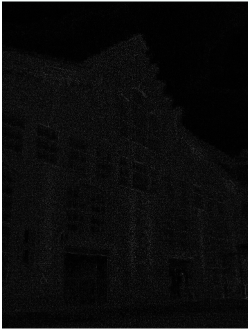
Rough analogy: digital images

600×800.tiff file
1400 KB

"full" N-body

.jpg file
223 KB

high order



$\Delta \approx 5\%$

$\Delta = 100\%$ means white noise difference image; zero means identical.

Expansion methods are like JPEG, lossy but flexible.

Theory

The Poisson equation is $\nabla^2 \Phi(\mathbf{r}) = 4\pi\rho(\mathbf{r})$.
The formal solution:

$$\Phi(\mathbf{r}) = -\int \frac{1}{|\mathbf{r} - \mathbf{r}'|} \rho(\mathbf{r}') \, \mathrm{d}^3\mathbf{r}'$$

— — — — — — — — — — — — — — — — — —

Theory

The Poisson equation is $\nabla^2\Phi(\mathbf{r}) = 4\pi\rho(\mathbf{r})$.

The formal solution:

Taylor series

$$\Phi(\mathbf{r}) = -\int \frac{1}{|\mathbf{r}-\mathbf{r}'|}\,\rho(\mathbf{r}')\,\mathrm{d}^3\mathbf{r}'$$

Theory

The Poisson equation is $\nabla^2\Phi(\mathbf{r}) = 4\pi\rho(\mathbf{r})$.
The formal solution:

Taylor series

$$\Phi(\mathbf{r}) = -\int \frac{1}{|\mathbf{r}-\mathbf{r}'|}\rho(\mathbf{r}')\,\mathrm{d}^3\mathbf{r}'$$

generalized **Fourier** series

Theory

The Poisson equation is $\nabla^2\Phi(\mathbf{r}) = 4\pi\rho(\mathbf{r})$.
The formal solution:

Taylor series

$$\Phi(\mathbf{r}) = -\int \frac{1}{|\mathbf{r}-\mathbf{r}'|}\rho(\mathbf{r}')\,\mathrm{d}^3\mathbf{r}'$$

generalized **Fourier** series

$$\Phi(\mathbf{r}) = \sum_k a_k \Phi_k(\mathbf{r})$$

In MEX you know the coefficients and numerically evaluate the functions; in SCF you pick a function basis in advance and need to calculate the coefficients from the density.

Theory

MEX has *two* sums while SCF has *three*. Thus:

$$\text{MEX} = \lim_{n\to\infty} \text{SCF}$$

However MEX requires the particles to be *sorted*. ☹

— — — — — — — — — — — — — — — — — —

Theory

MEX has *two* sums while SCF has *three*. Thus:

$$\text{MEX} = \lim_{n\to\infty} \text{SCF}$$

However MEX requires the particles to be *sorted*. ☹

— — — — — — — — — — — — — — — — — —

MEX is ‘accurate’ in the radial direction.

Decompose the mass density $\rho(\mathbf{r}) = \bar{\rho}(r) + \tilde{\rho}(r, \theta, \phi)$ to a *spherical average* and the non-spherical deviation.

$$\Phi(\mathbf{r}) = \Phi_0(r) + \sum_{l=1}^{\infty} \Phi_l(r, \theta, \phi)$$

Theory

MEX has *two* sums while SCF has *three*. Thus:

$$\text{MEX} = \lim_{n\to\infty} \text{SCF}$$

However MEX requires the particles to be *sorted*. ☹

— — — — — — — — — — — — — — — — — —

MEX is 'accurate' in the radial direction.

Decompose the mass density $\rho(\mathbf{r}) = \bar{\rho}(r) + \tilde{\rho}(r, \theta, \phi)$
to a *spherical average* and the non-spherical deviation.

$$\Phi(\mathbf{r}) = \Phi_0(r) + \sum_{l=1}^{\infty} \Phi_l(r, \theta, \phi)$$

Theory

MEX has *two* sums while SCF has *three*. Thus:

$$\text{MEX} = \lim_{n\to\infty} \text{SCF}$$

However MEX requires the particles to be *sorted*. ☹

— — — — — — — — — — — — — — — — — —

MEX is 'accurate' in the radial direction.

Decompose the mass density $\rho(\mathbf{r}) = \bar{\rho}(r) + \tilde{\rho}(r, \theta, \phi)$ to a *spherical average* and the non-spherical deviation.

$$\Phi(\mathbf{r}) = \Phi_0(r) + \sum_{l=1}^{\infty} \Phi_l(r, \theta, \phi)$$

In SCF need to *choose* the basis set wisely. ☹

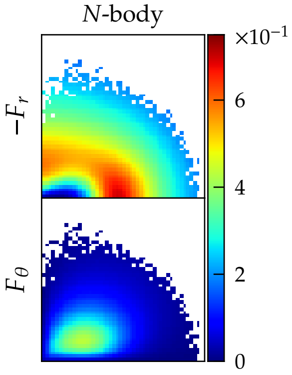
Implementation

Everything but i/o is done on the GPU!

"Thrust is a parallel algorithms library which resembles the C++ Standard Template Library (STL)"

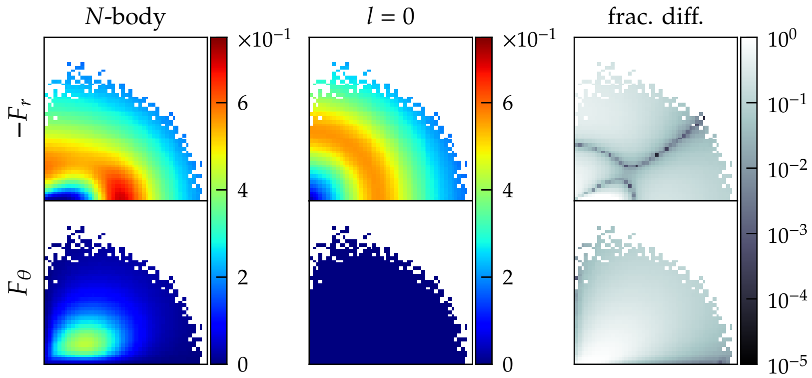
- Built-in subroutines (sort, prefix sum, reduce, transform. . .)
- Transparent GPU programming in C++ ~~(no CUDA calls)~~
- Change to OpenMP with a compilation flag.

How many multipoles?



Rotating King model with $N = 10^6$, $W_0 = 6$ and $\omega_0 = 1.8$ (fast rotation)

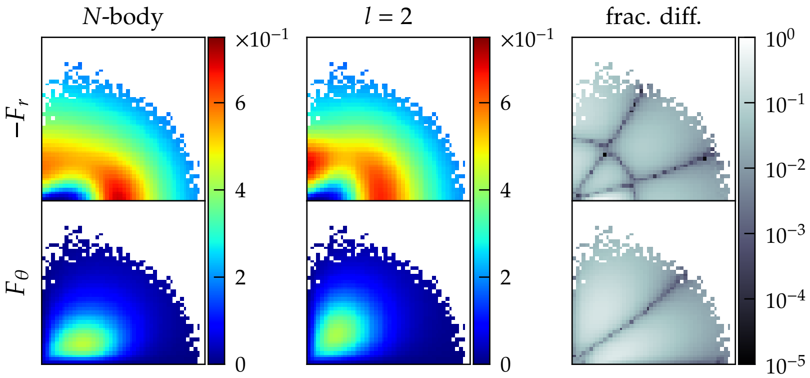
How many multipoles?



Rotating King model with $N = 10^6$, $W_0 = 6$ and $\omega_0 = 1.8$ (fast rotation)

Just the monopole

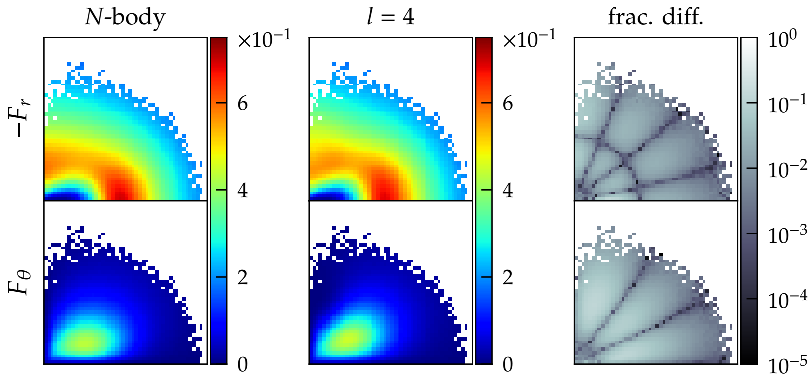
How many multipoles?



Rotating King model with $N = 10^6$, $W_0 = 6$ and $\omega_0 = 1.8$ (fast rotation)

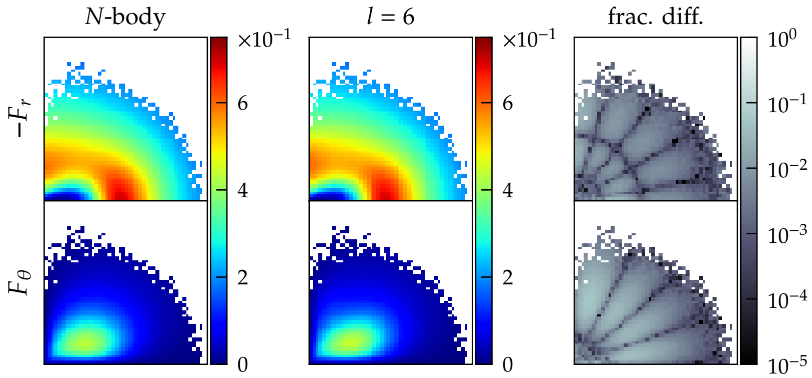
Monopole and quadrupole

How many multipoles?



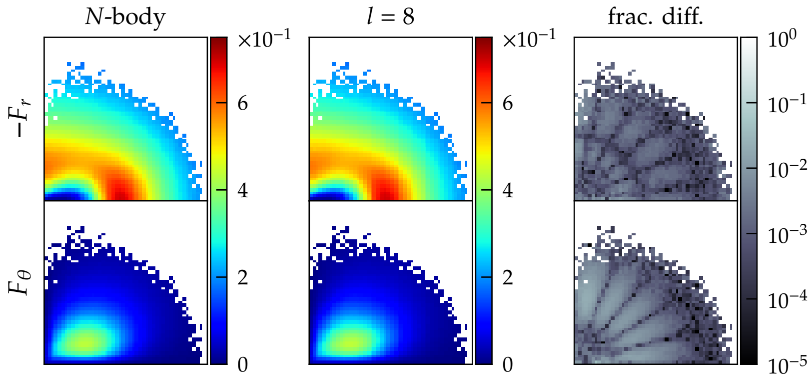
Rotating King model with $N = 10^6$, $W_0 = 6$ and $\omega_0 = 1.8$ (fast rotation)

How many multipoles?



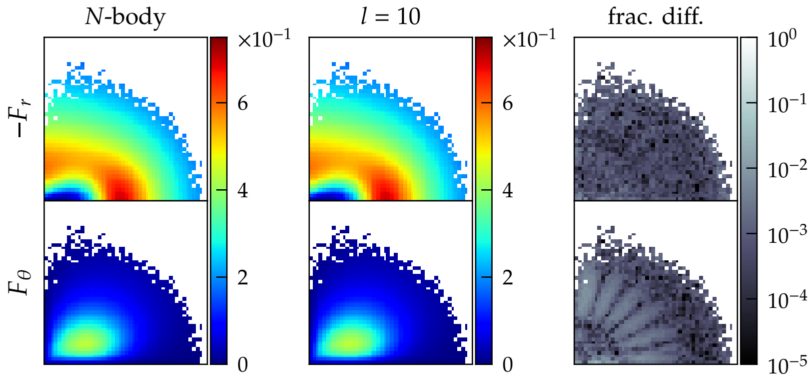
Rotating King model with $N = 10^6$, $W_0 = 6$ and $\omega_0 = 1.8$ (fast rotation)

How many multipoles?



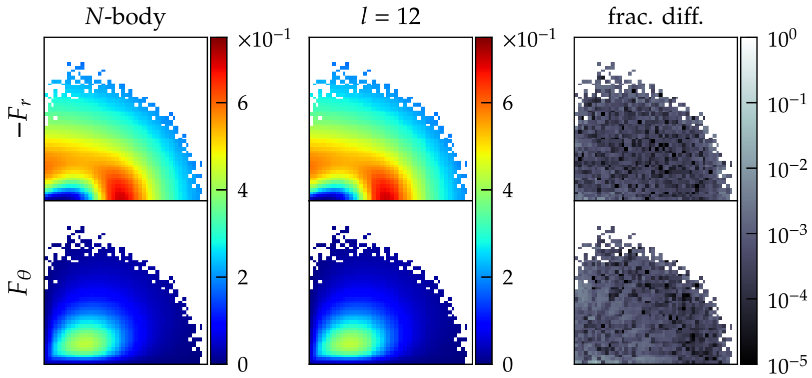
Rotating King model with $N = 10^6$, $W_0 = 6$ and $\omega_0 = 1.8$ (fast rotation)

How many multipoles?



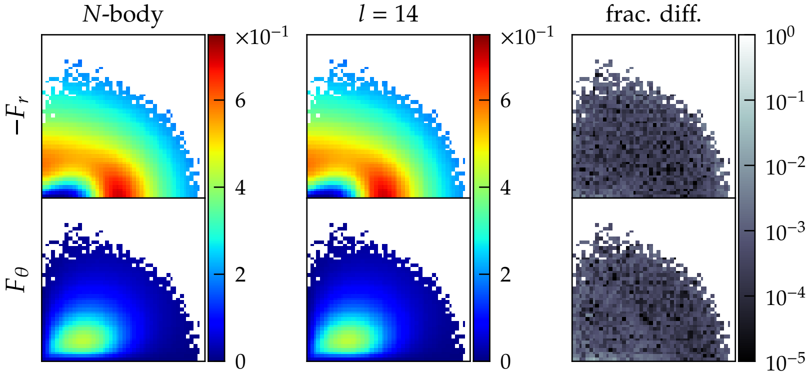
Rotating King model with $N = 10^6$, $W_0 = 6$ and $\omega_0 = 1.8$ (fast rotation)

How many multipoles?



Rotating King model with $N = 10^6$, $W_0 = 6$ and $\omega_0 = 1.8$ (fast rotation)

How many multipoles?

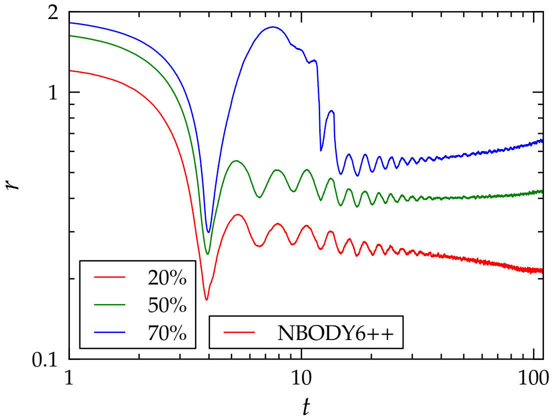


Rotating King model with $N = 10^6$, $W_0 = 6$ and $\omega_0 = 1.8$ (fast rotation)

Up to sedecmiliatrecentioctogintaquadrupole

First test (monopole only)

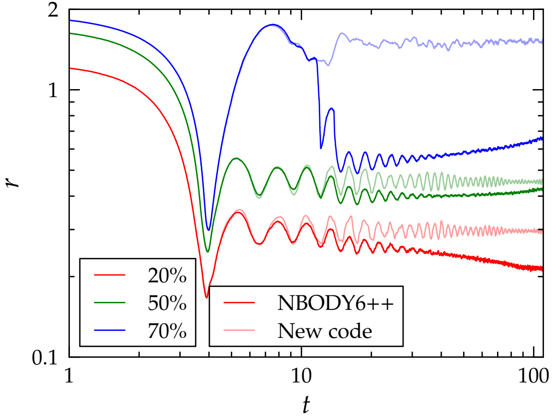
Spherical collapse



Uniform density sphere ($N = 6 \times 10^4$), virial factor $Q = 0.1$

First test (monopole only)

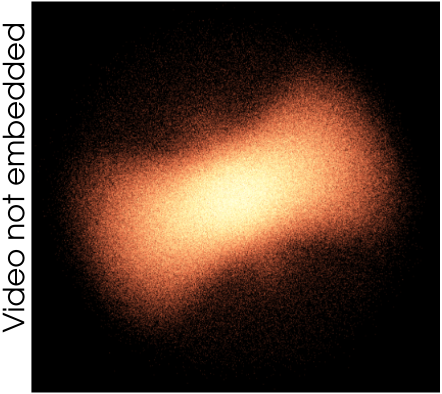
Spherical collapse



Uniform density sphere ($N = 6 \times 10^4$), virial factor $Q = 0.1$

More interesting test

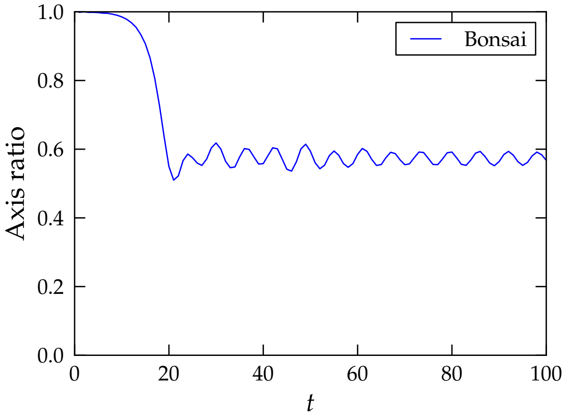
Radially anisotropic spherical collapse



Special DF (Polyachenko et al. 2013) with $N = 10^6$

More interesting test

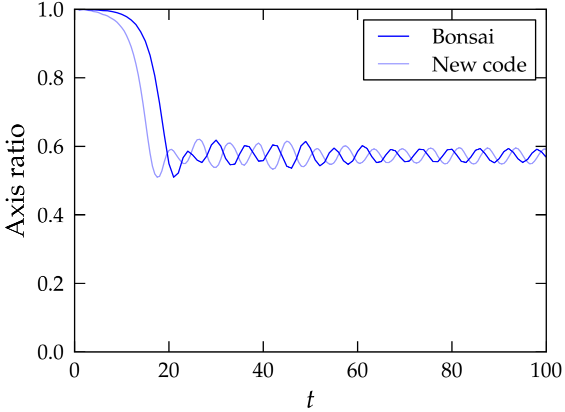
Radially anisotropic spherical collapse



Comparison between a tree code and MEX

More interesting test

Radially anisotropic spherical collapse



Comparison between a tree code and MEX

Summary

Status report

We have very good MEX & SCF potential solvers for GPUs.

The following extensions are pending:

- Collisional particles
- Good time stepping
- MPI parallelization

- Amuse module?